Ideal Operational Amplifiers

MES Institute of Microelectronic Systems

Overview

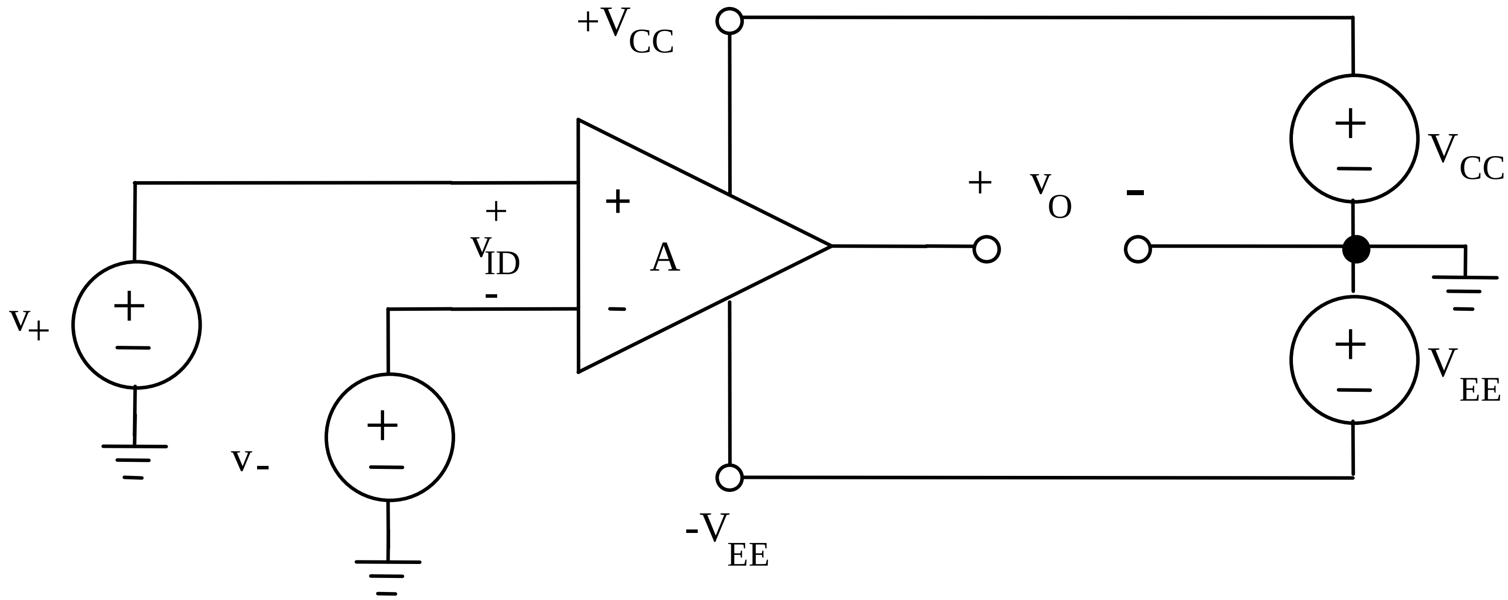
- Terminology and history
- Differential Amplifier
- Ideal Operational Amplifier
- Analysis of Circuits Containing Ideal Operational Amplifiers
  - Inverting and Noninverting Amplifier
  - Voltage Follower
  - Summing Amplifier, Difference Amplifier, Instrumentation-Amplifier Configuration, Low-Pass Filter, Integrator
- Circuits Using Positive Feedback
  - The Comparator and Schmitt Trigger
  - The Astable Multivibrator
  - The Monostable Multivibrator

Terminology and History

- The **operational amplifier** or **op amp** is a fundamental building block of analog circuit design.
- The name "operational amplifier" originates from the use of this type of amplifier to perform specific electronic circuit functions or operations, such as scaling, summation, and integration.

- The m**A-709**, introduced by Fairchild Semiconductors in 1965, was one of the first widely used general-purpose IC operational amplifiers.
- The now classic m**A-741** amplifier by Fairchild Semiconductors, which appeared in the late 1960s, is a robust amplifier with excellent characteristics for most general-purpose applications.

Differential Amplifier (I)



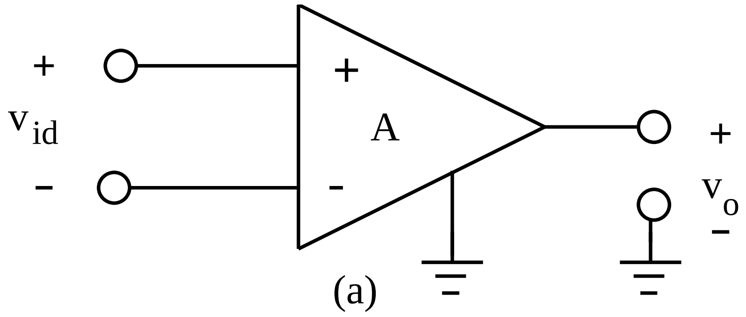
The differential amplifier, including power supplies

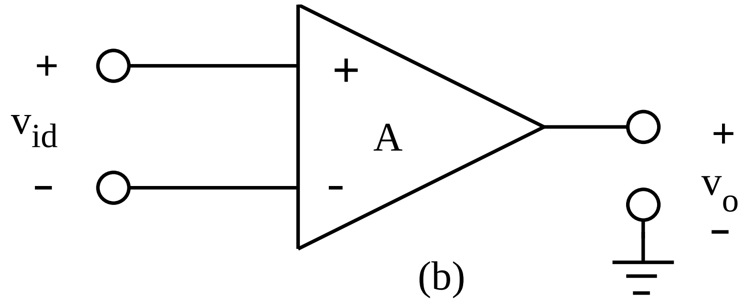
In most applications, $V_{CC} \geq 0$ and $-V_{EE} \leq 0$, and the voltages are often symmetric — that is, ±5 V, ±12 V, ±15 V, and so on.

These power supply voltages limit the output voltage range:

$$-V_{EE} \leq v_O \leq V_{CC}$$

Differential Amplifier (II)

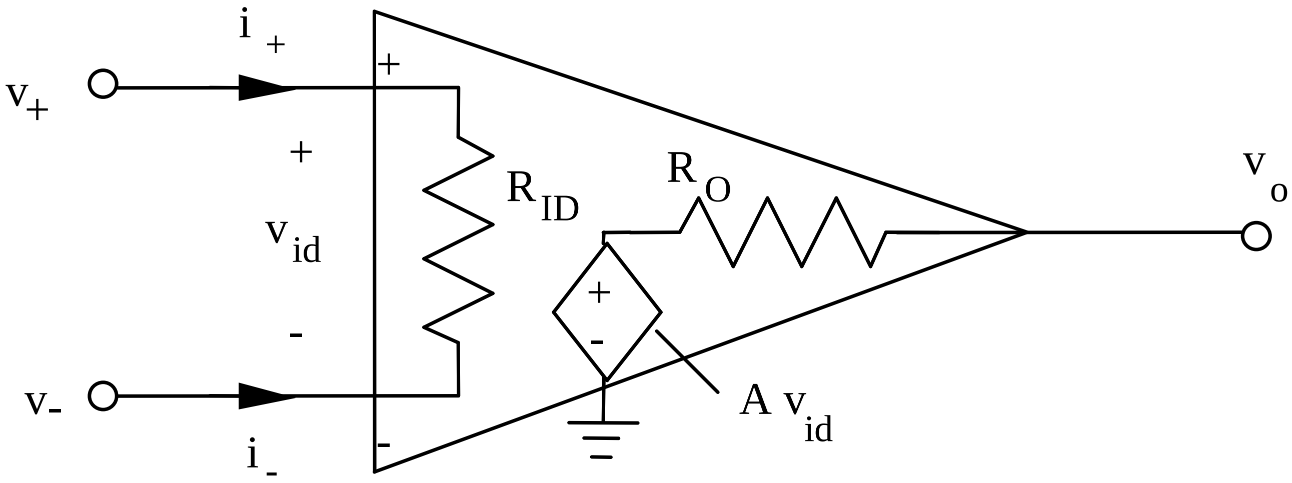




(a) Amplifier without power supplies explicitly included

(b) Differential amplifier with implied ground connections

Differential Amplifier (III)



Differential amplifier

A = voltage gain (open-ciruit voltage gain)

v_{id} = $(v_+ - v_-)$ = differential input signal voltage

R_{ID} = amplifier input resistance

R_O = amplifier output resistance

Differential Amplifier (IV)

The signal voltage developed at the output of the amplifier is in phase with the voltage applied to the + input terminal and 180° out of phase with the signal applied to the — input terminal.

The v_+ and v_- terminals are therefore referred to as the **noninverting input** and the **inverting input**, respectively.

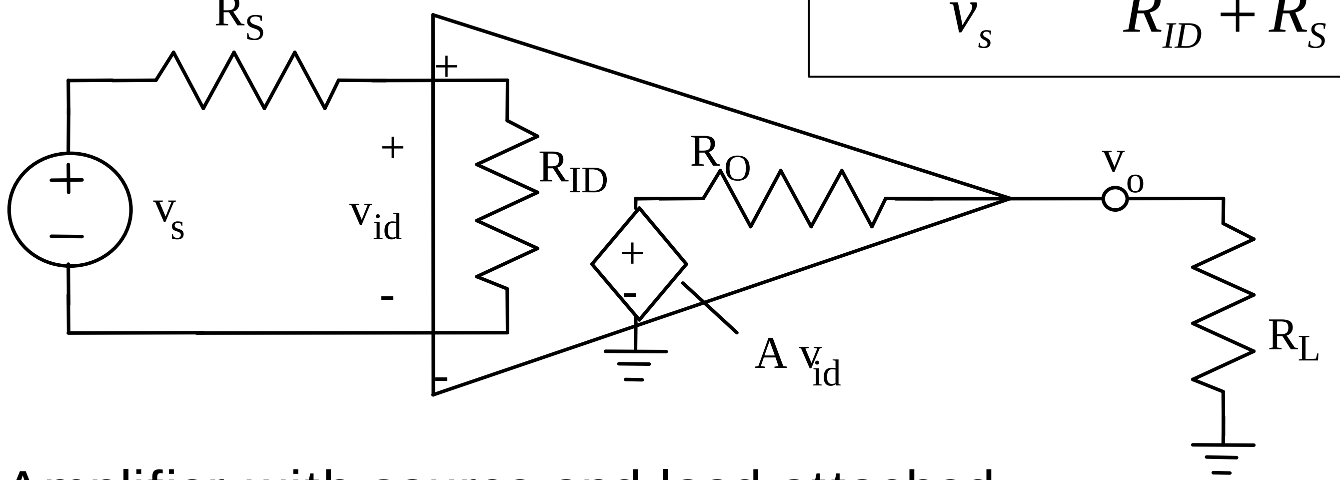
Differential Amplifier (V)

In a typical application, the amplifier is driven by a signal source having a Thévenin equivalent voltage v_s and resistance R_s and is connected to a load R_L:

$$v_o = A v_{id} \frac{R_L}{R_o + R_L}$$

$$v_{id} = v_s \frac{R_{ID}}{R_{ID} + R_S}$$

$$\Rightarrow \boxed{A_V = \frac{v_o}{v_s} = A \frac{R_{ID}}{R_{ID} + R_S} \frac{R_L}{R_o + R_L}}$$



Amplifier with source and load attached

Ideal Differential Amplifier

An ideal differential amplifier would produce an output that depends only on the voltage difference v_{id} between its two input terminals, and this voltage would be independent of source and load resistances.

This behavior can be achieved if the input resistance of the amplifier is infinite and the output resistance is zero:

$$v_o = Av_{id} \quad \text{or} \quad A_V = \frac{v_o}{v_{id}} = A$$

A is referred to as either the **open-circuit voltage gain** or **open-loop gain** of the amplifier and represents the maximum voltage gain available from the device.

Ideal Operational Amplifier (I)

An ideal operational amplifier is an ideal differential amplifier with infinite voltage gain:

$$R_{ID} = \infty$$

$$R_o = 0$$

$$A = \infty$$

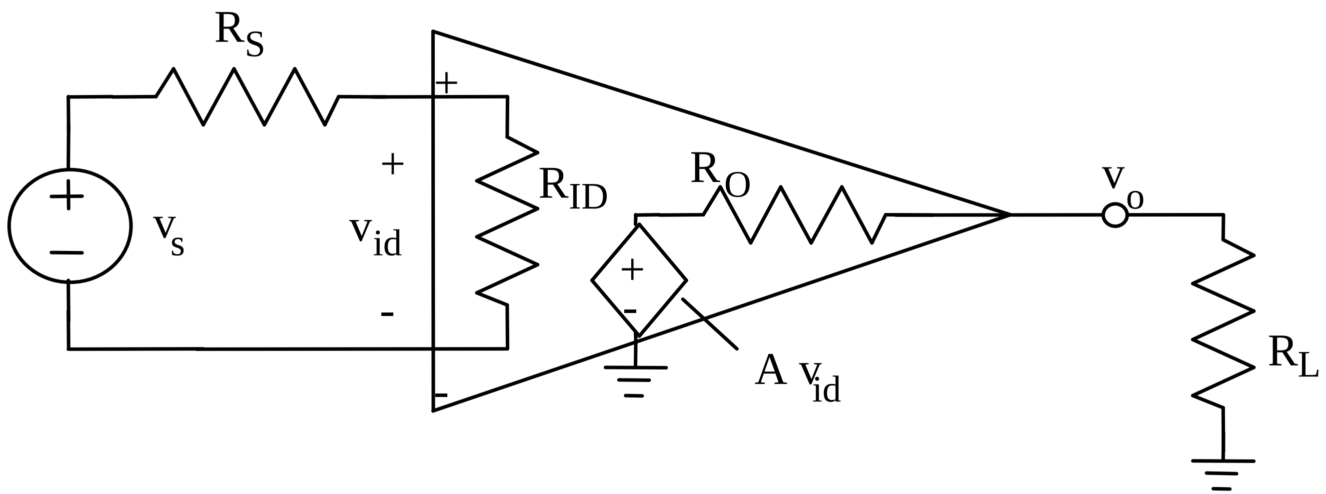
Infinite gain leads to the first central assumption in analyzing circuits containing op amps:

$$v_{ID} = \frac{v_o}{A} \quad \Rightarrow \quad \boxed{\lim_{A \to \infty} v_{ID} = 0}$$

If A is infinite, then the input voltage v_{id} will be zero for any finite output voltage: $v_{ID} = 0$

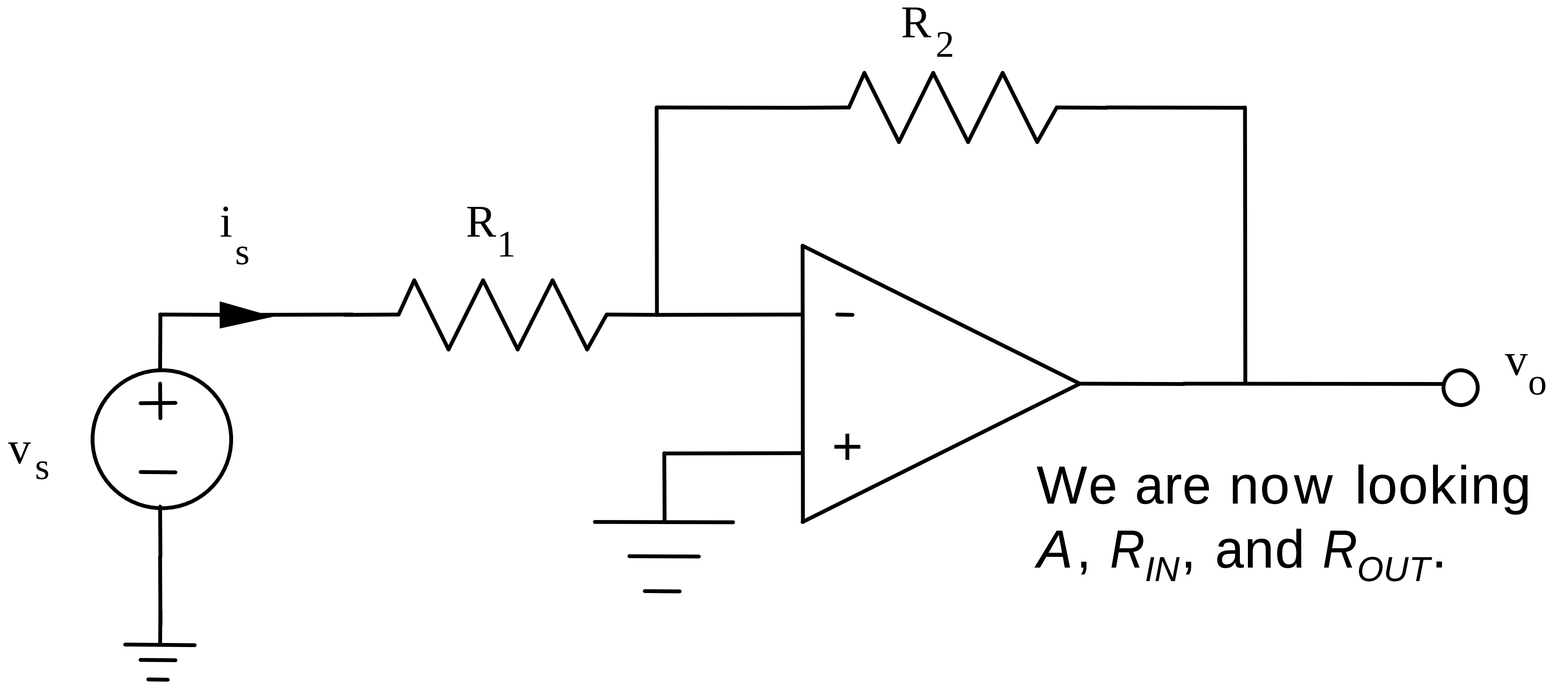
Ideal Operational Amplifier (II)

The second central assumption is caused by the infinite input resistance R_{ID} that forces the two input currents i_+ and i_- to be zero: $i_+ = 0 \ and \ i_- = 0$



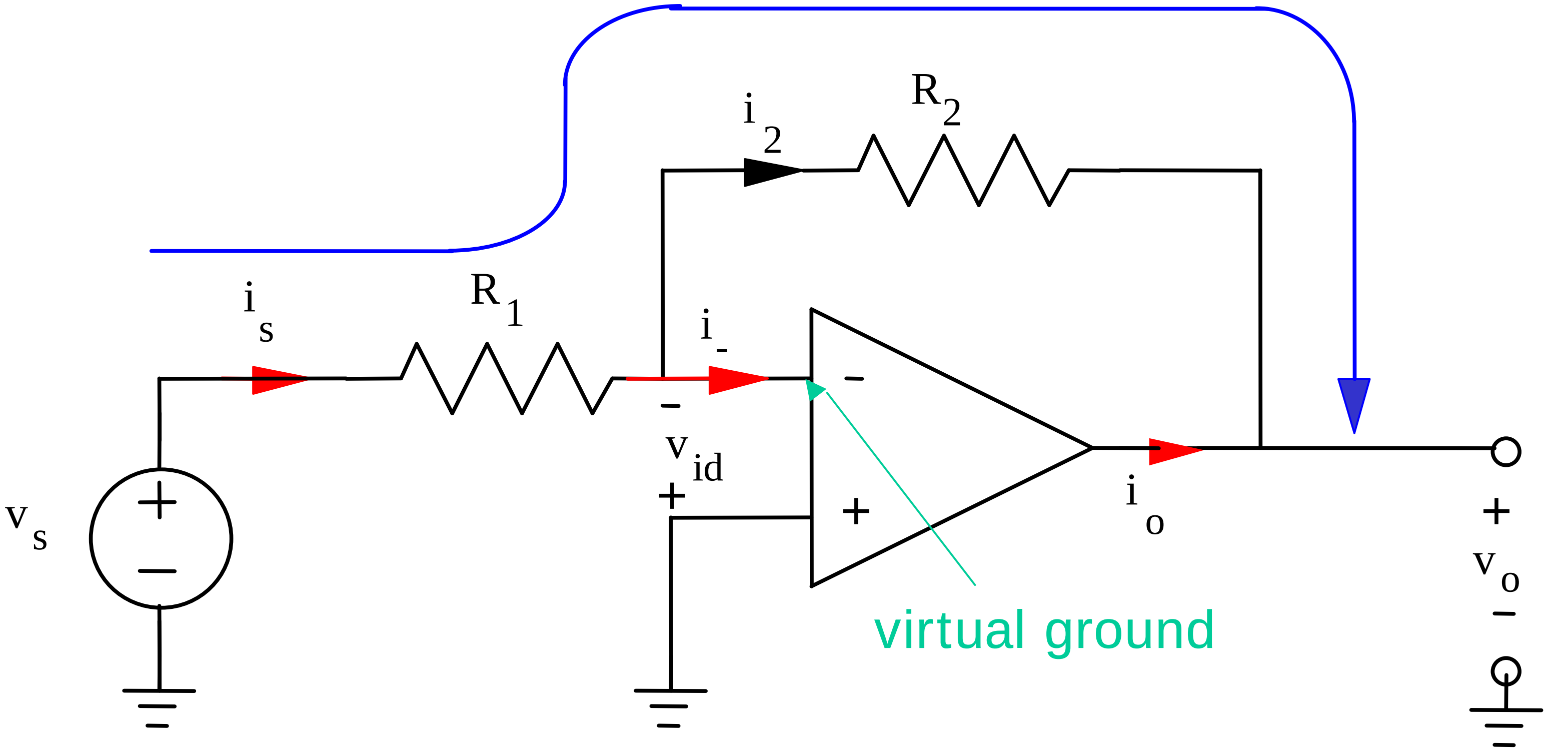
The Inverting Amplifier (I)

An inverting amplifier circuit is built by grounding the positive input of the operational amplifier and connecting resistors R_1 and R_2, called the **feedback network**, between the inverting input and the signal source and amplifier output node, respectively.



We are now looking for A, R_{IN}, and R_{OUT}.

The Inverting Amplifier (II)



The Inverting Amplifier: Voltage Gain

We begin by determining the voltage gain.

$$v_s - i_s R_1 - i_2 R_2 - v_o = 0 \qquad \Big|\; i_s = i_- + i_2 \;\Rightarrow\; i_s = i_2$$

$$v_s - i_s R_1 - i_s R_2 - v_o = 0$$

$$\Big|\; v_{id} = v_+ - v_- = 0$$

$$\Rightarrow v_+ = 0 \Rightarrow v_- = 0$$

$$i_s = \frac{v_s - v_-}{R_1}$$

$$\Rightarrow \quad i_s = \frac{v_s}{R_1}$$

$$-v_s \frac{R_2}{R_1} - v_o = 0$$

$$A_V = \frac{v_o}{v_s} \qquad \Rightarrow \boxed{A_V = -\frac{R_2}{R_1}}$$

The Inverting Amplifier: Input Resistance

The input resistance R_{IN} of the overall amplifier is found directly from the last slide:

$$i_s = \frac{v_s}{R_1}$$

$$\Rightarrow \boxed{R_{IN} = \frac{v_s}{i_s} = R_1}$$

The Inverting Amplifier: Output Resistance

The output resistance R_{OUT} is the Thévenin equivalent resistance; it is found by applying a test signal current (or voltage) source to the output of the amplifier and determining the voltage (or current). All other independent voltage and current sources in the circuit must be turned off, and v_s is set zo zero.

$$R_{OUT} = \frac{v_x}{i_x}$$

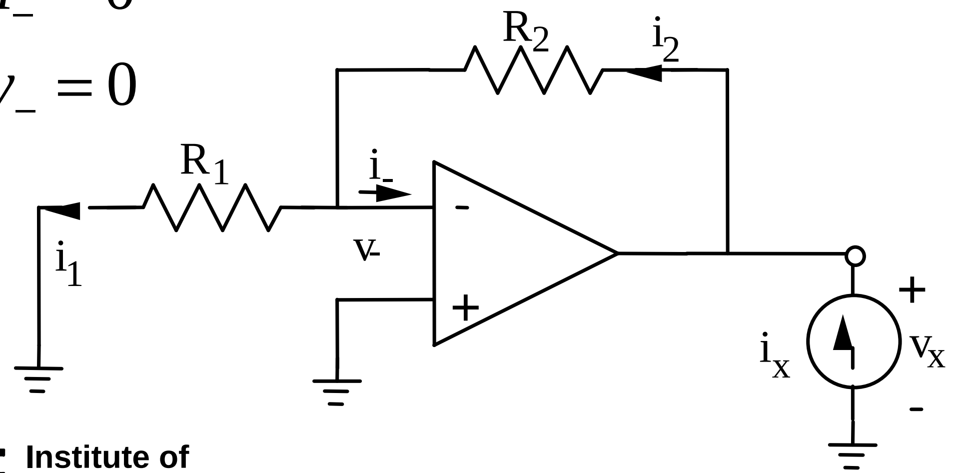
$$v_x = i_2 R_2 + i_1 R_1 \;\middle|\; i_1 = i_2 \;\text{ because }\; i_- = 0$$

$$v_x = i_1 (R_2 + R_1) \;\middle|\; i_1 = 0 \;\text{ because }\; v_- = 0$$

Thus, $v_x = 0$ independent of

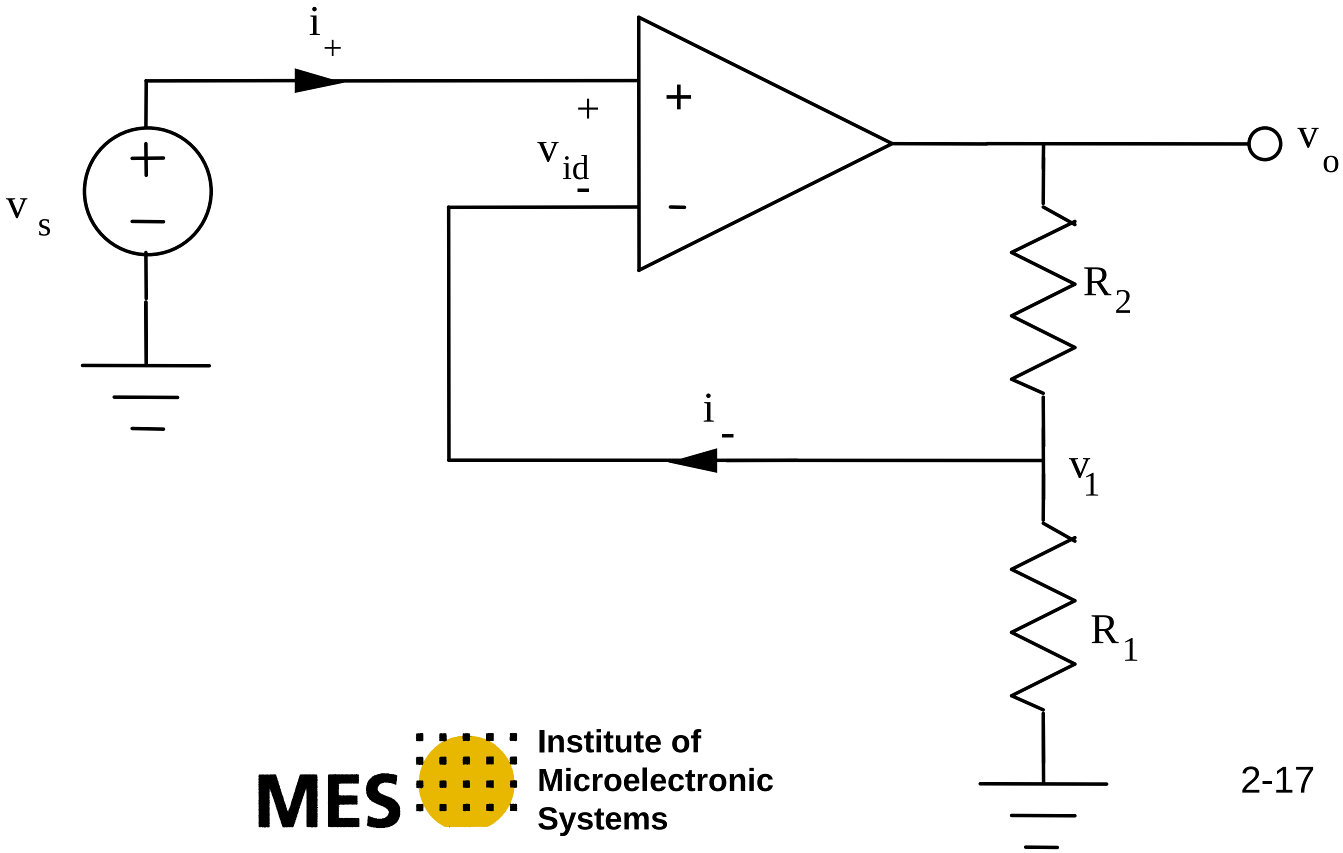
the value of i_x, and

$$\boxed{R_{OUT} = 0}$$



The Noninverting Amplifier

The operational amplifier can also be used to construct a **noninverting amplifier**. The input signal is applied to the positive (noninverting) input terminal, and a portion of the output signal is fed back to the negative input terminal.



The Noninverting Amplifier: Voltage Gain

$$i_- = 0$$

$$v_1 = v_o \frac{R_1}{R_1 + R_2}$$

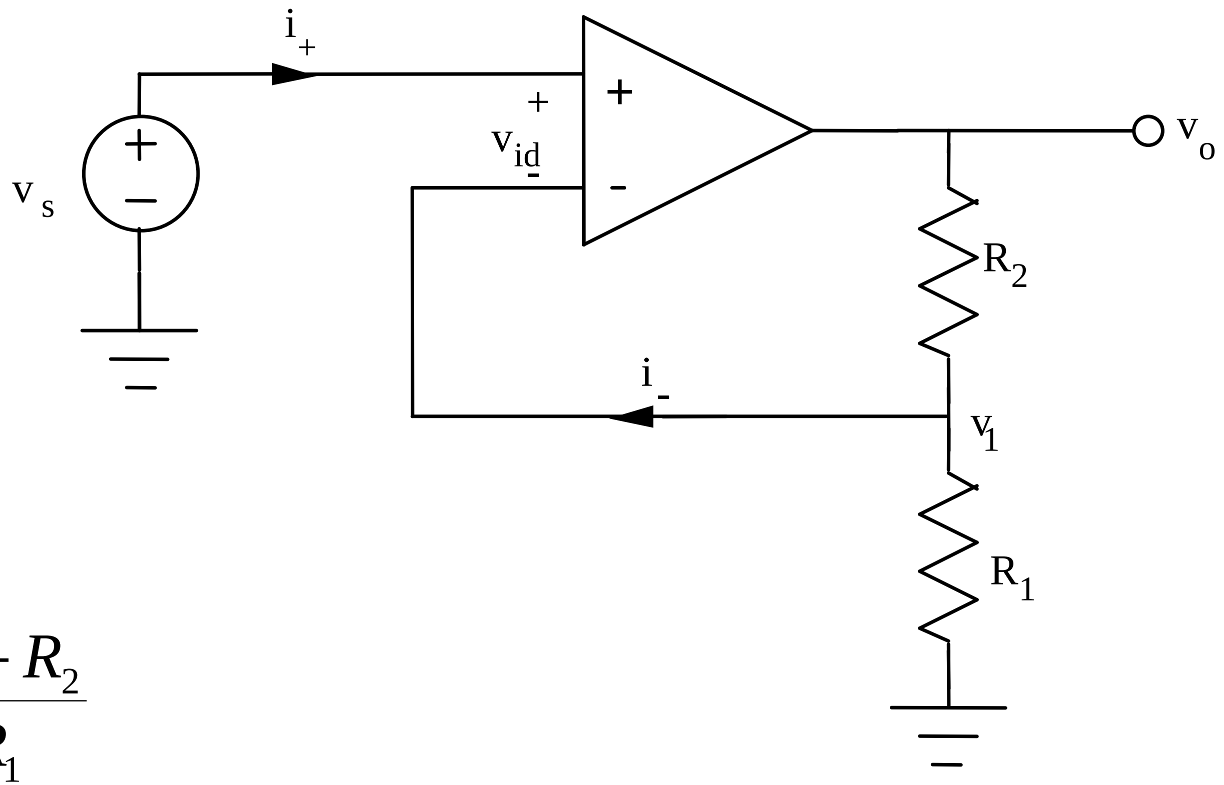
$$v_s - v_{id} = v_1$$

$$v_{id} = 0 \Rightarrow v_s = v_1$$

$$v_o = v_s \frac{R_1 + R_2}{R_1}$$

$$\Rightarrow A_V = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1}$$

$$\Rightarrow \boxed{A_V = 1 + \frac{R_2}{R_1}}$$



Note that the gain is positive and must be greater than or equal 1.

The Noninverting Amplifier: Input and Output Resistance

$$R_{IN} = \frac{v_s}{i_+}$$

$$\boxed{R_{IN} = \infty} \text{ because } i_+ = 0$$

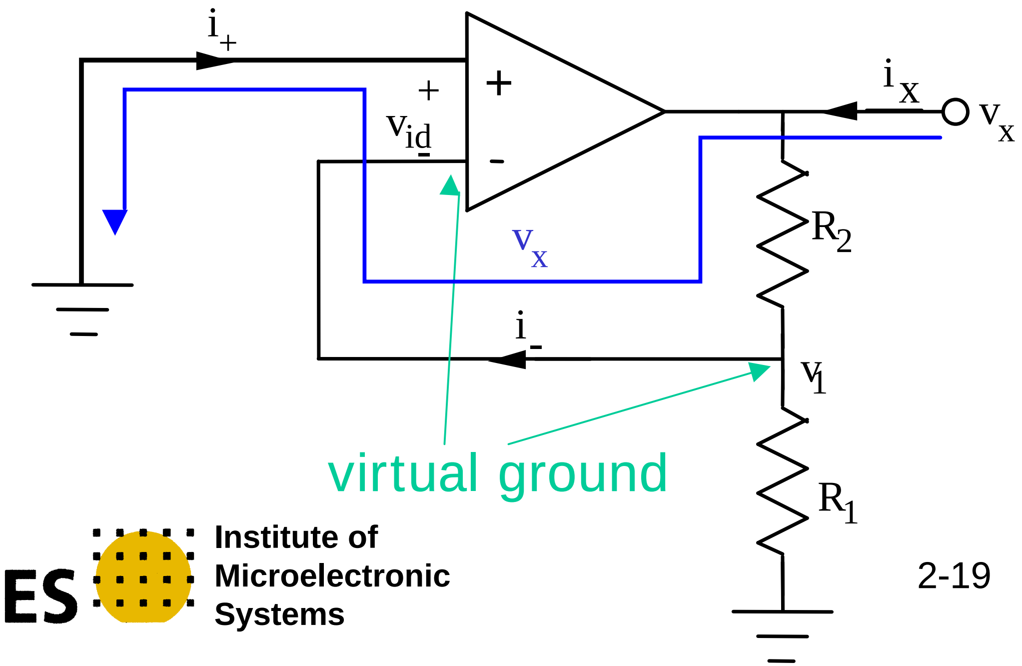
To find the output resistance, a test current is applied to the output terminal and the source v_s is set to 0.

$$R_{OUT} = \frac{v_x}{i_x}$$

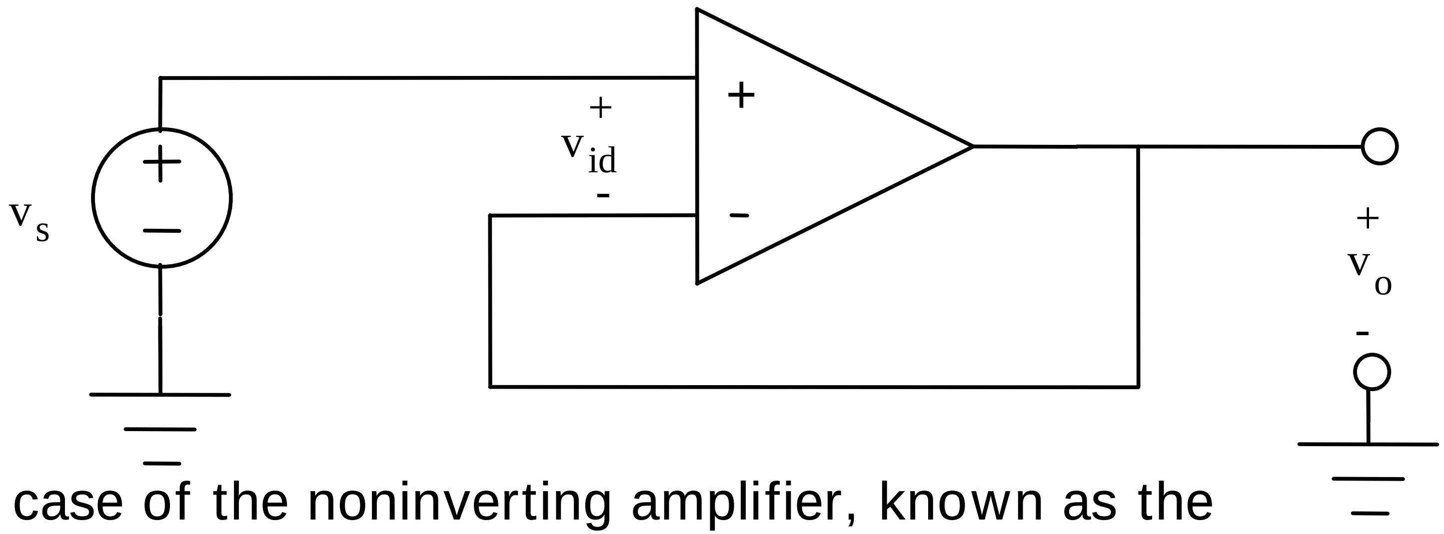
$$v_{id} = 0$$

$$v_x = R_2 i_-$$

$$\boxed{R_{OUT} = 0} \text{ because } i_- = 0$$



Unity-Gain Buffer, or Voltage Follower (I)



A special case of the noninverting amplifier, known as the **unity-gain buffer**, or **voltage follower** is shown above.

The value of R_1 is infinite and that of R_2 is zero.

We find for the voltage gain: $v_s - v_{id} = v_o$ | $v_{id} = 0$

or $v_o = v_s$

$$\Rightarrow \boxed{A_V = 1}$$

Unity-Gain Buffer, or Voltage Follower (II)

Why is such an amplifier useful?

The ideal unity-gain buffer provides a gain of 1 with infinite input resistance and zero output resistance and therefore provides a tremendous **impedance-level transformation** while maintaining the level of the signal voltage.

The ideal unity-gain buffer does not require any input current, yet can drive any desired load resistance without loss of signal voltage. Thus, the unity-gain buffer is found in many sensor and data acqusition applications.

Summary of Ideal Inverting and Noninverting Amplifier Characteristics

	Inverting Amplifier	Non-Inverting Amplifier
Voltage Gain A_V	$-\frac{R_2}{R_1}$	$1+\frac{R_2}{R_1}$
Input Resistance R_{IN}	R_1	∞
Output Resistance R_{OUT}	0	0

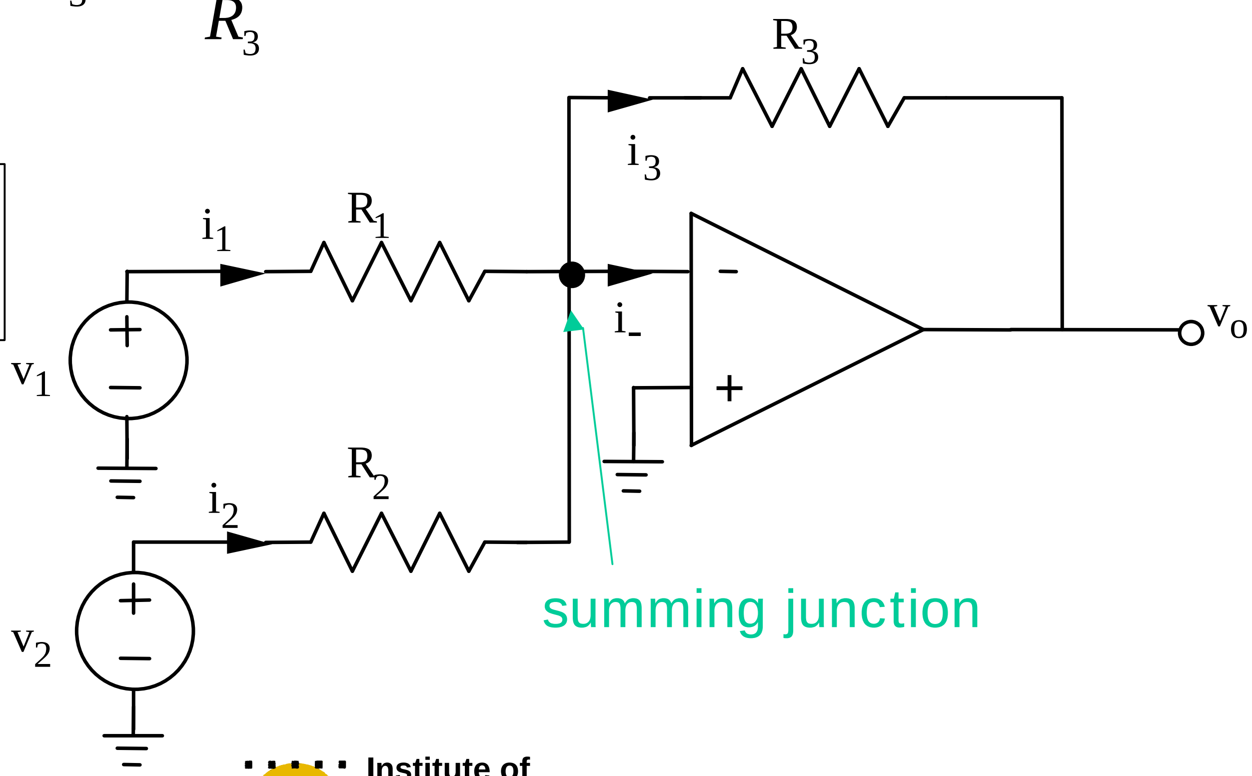
The Summing Amplifier

Two input sources v_1 and v_2 are connected to the inverting input through resistors R_1 and R_2.

$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_2}{R_2} \qquad i_3 = -\frac{v_o}{R_3}$$

$$i_- = 0 \qquad i_3 = i_1 + i_2$$

$$\boxed{v_o = -\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_2}$$



Any number of inputs can be put to the summing junction.

The Difference Amplifier

$$v_o = v_- - i_2 R_2 = v_- - i_1 R_2$$

$$i_1 = \frac{v_1 - v_-}{R_1}$$

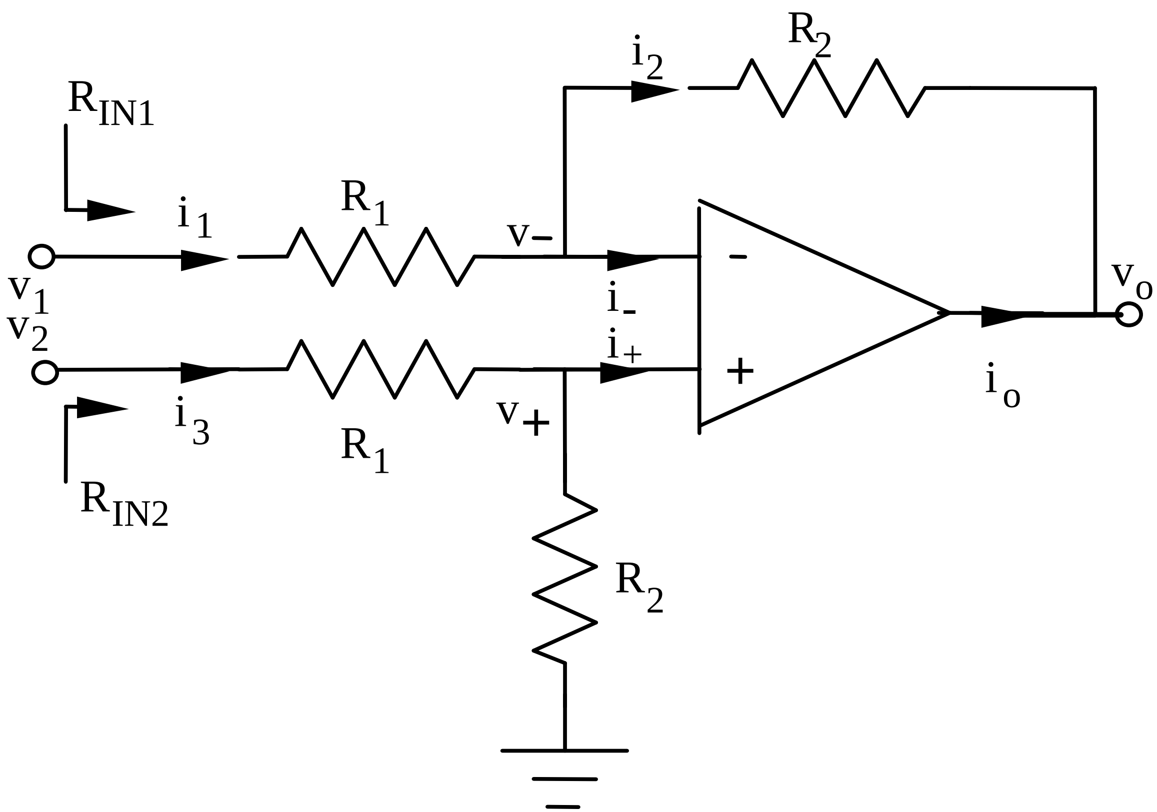
$$v_o = v_- - \frac{R_2}{R_1}(v_1 - v_-)$$

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_- - \frac{R_2}{R_1} v_1$$

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_+ - \frac{R_2}{R_1} v_1$$

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_2}{R_1 + R_2}\right) v_2 - \frac{R_2}{R_1} v_1$$

$$\boxed{v_o = \left(-\frac{R_2}{R_1}\right)(v_1 - v_2)}$$



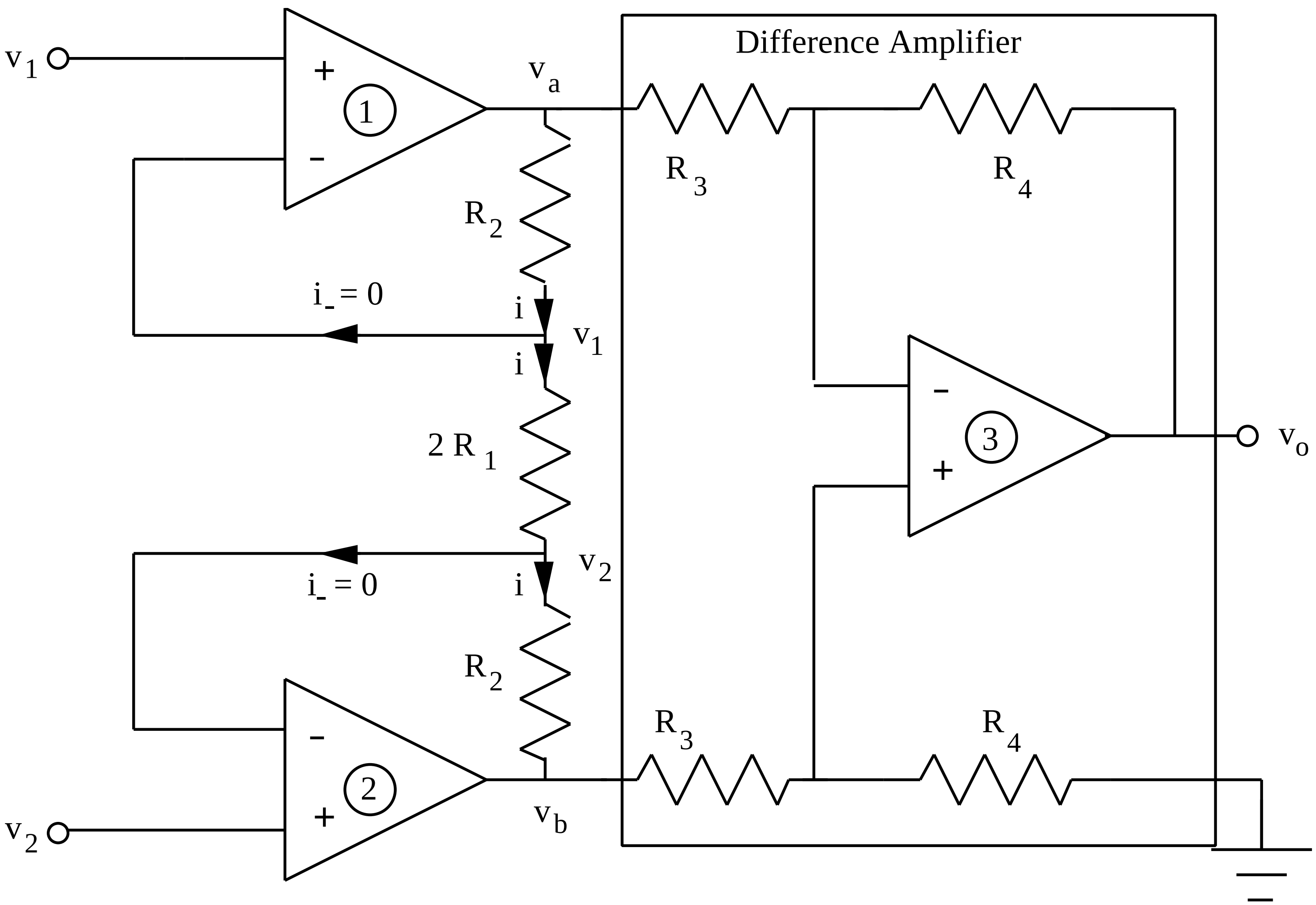
The operational amplifier may itself be used in a difference amplifier configuration, which amplifies the difference between two input signals.

The Instrumentation-Amplifier Configuration (I)

We often need to amplify the difference in two signals but cannot use the difference amplifier presented on the last slide, because its input resistance is too low.

In such a case, we can combine two noninverting amplifiers with a difference amplifier to form the high-performance composite **instrumentation amplifier**.

The Instrumentation-Amplifier Configuration (II)



The Instrumentation-Amplifier Configuration (III)

$$v_o = \left(-\frac{R_4}{R_3}\right)(v_a - v_b)$$

$$v_b = v_a - iR_2 - i(2R_1) - iR_2$$

or

$$v_a - v_b = 2i(R_1 + R_2)$$

$$i = \frac{v_1 - v_2}{2R_1}$$

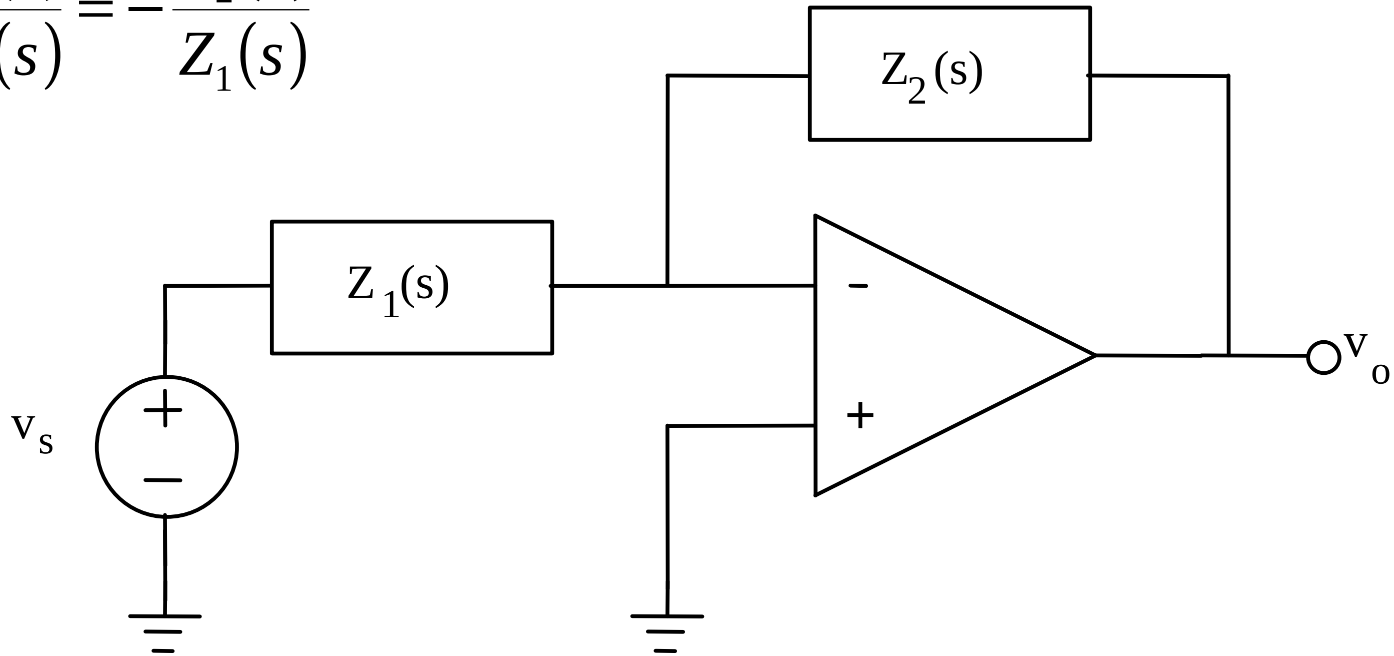
$$\boxed{v_o = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)(v_1 - v_2)}$$

The input resistance presented to both input sources is infinite because the input current to op amps is zero, and the output resistance is forced to zero by the difference

General Feedback Network

The general case of the inverting configuration with passive feedback is shown on this slide. Resistors R_1 and R_2 have been replaced by general impedances $Z_1(s)$ and $Z_2(s)$, which may now be a function of frequency.

$$A_V(s) = \frac{V_o(s)}{V_s(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



Low-Pass Filter (I)

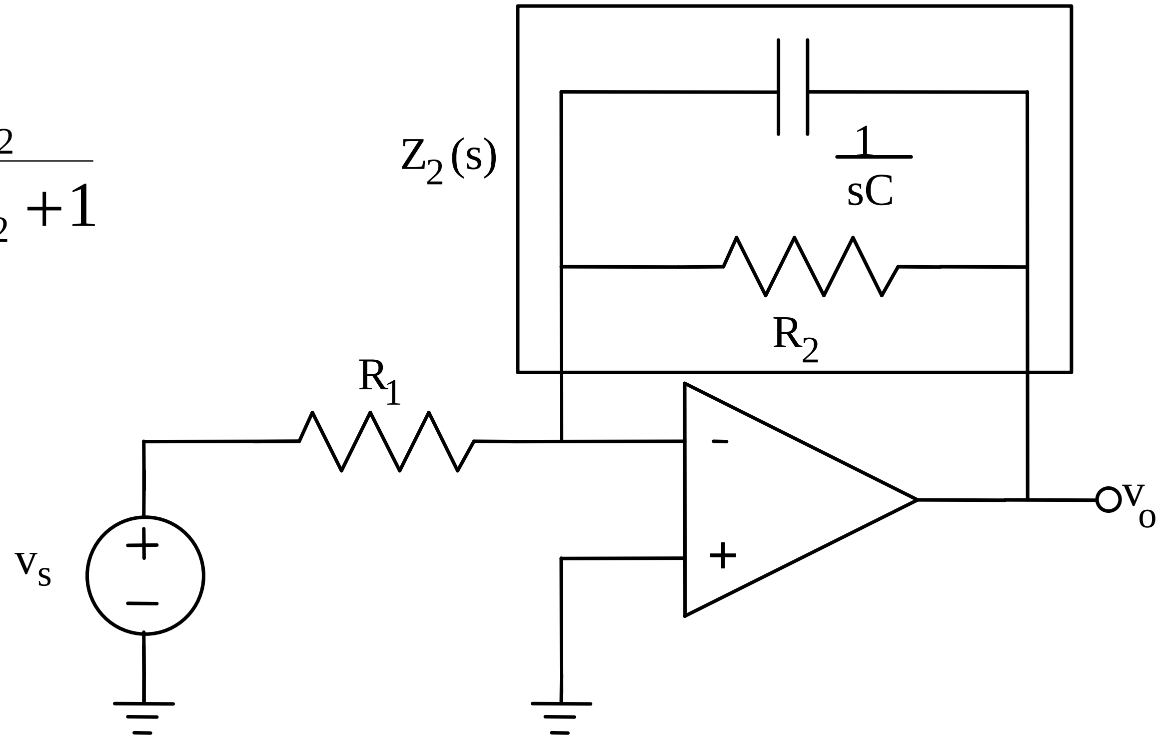
$$Z_1(s) = R_1 \quad \text{and}$$

$$Z_2(s) = \frac{R_2 \dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}} = \frac{R_2}{sCR_2 + 1}$$

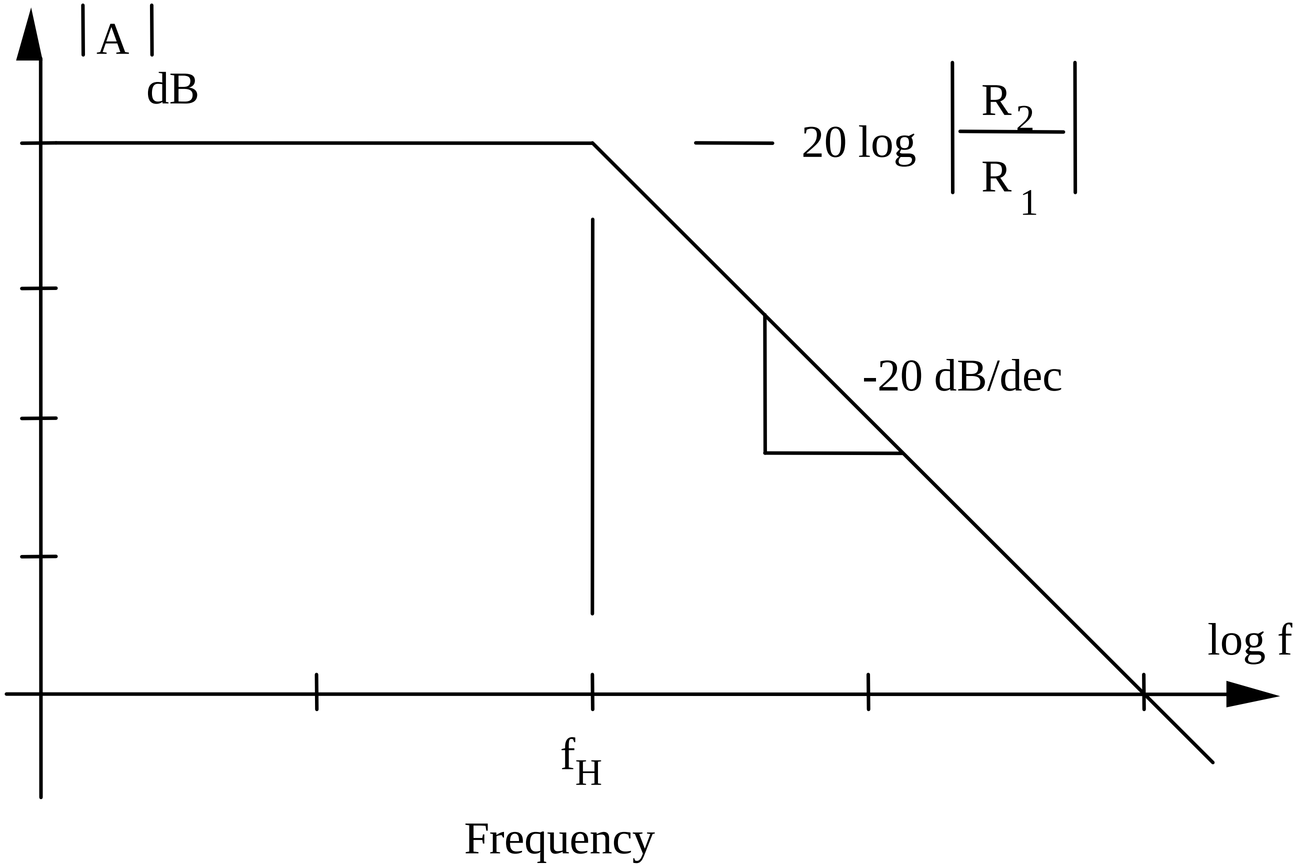
$$A_V(s) = -\frac{R_2}{R_1}\frac{1}{sCR_2 + 1}$$

$$\boxed{A_V(s) = -\frac{R_2}{R_1}\frac{1}{\dfrac{s}{\omega_H} + 1}}$$

where $\omega_H = 2\pi f_H = \dfrac{1}{R_2 C}$



Low-Pass Filter (II)



Integrator

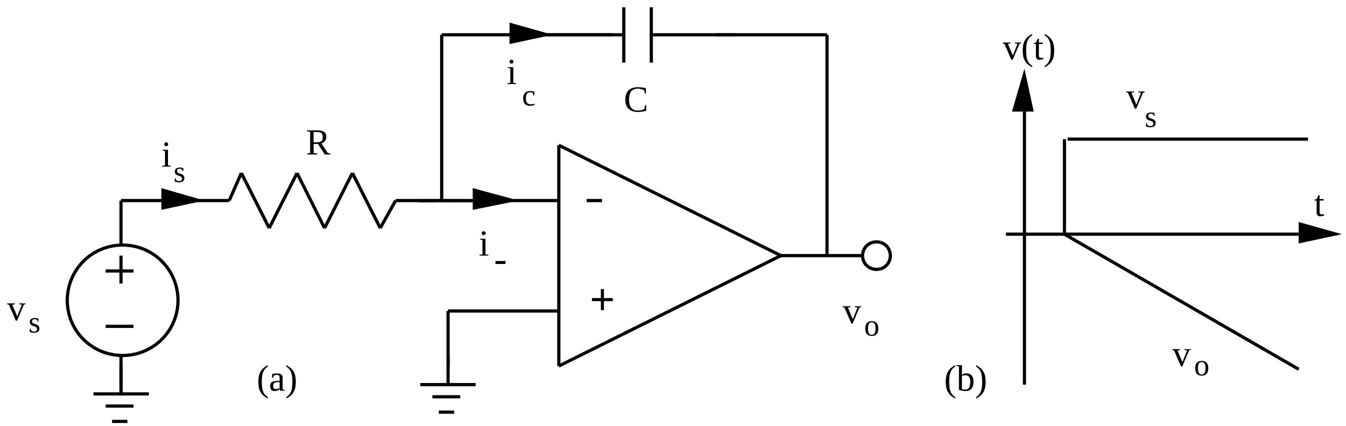
This circuit provides an opportunity to explore op amp circuit analysis in the time domain.

$$i_s = \frac{v_s}{R} \quad \text{and} \quad i_c = -C\frac{dv_o}{dt}$$

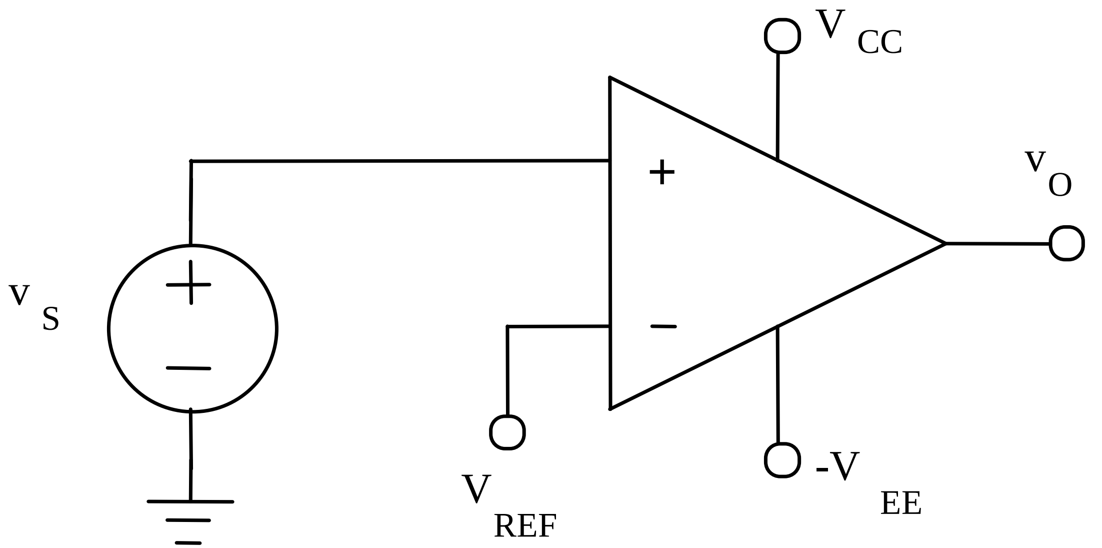
$$i_- = 0 \quad \Rightarrow i_c = i_s$$

$$\int dv_o = \int -\frac{1}{RC} v_s dt$$

$$v_o(t) = -\frac{1}{RC}\int_0^t v_s(t)dt + v_o(0)$$



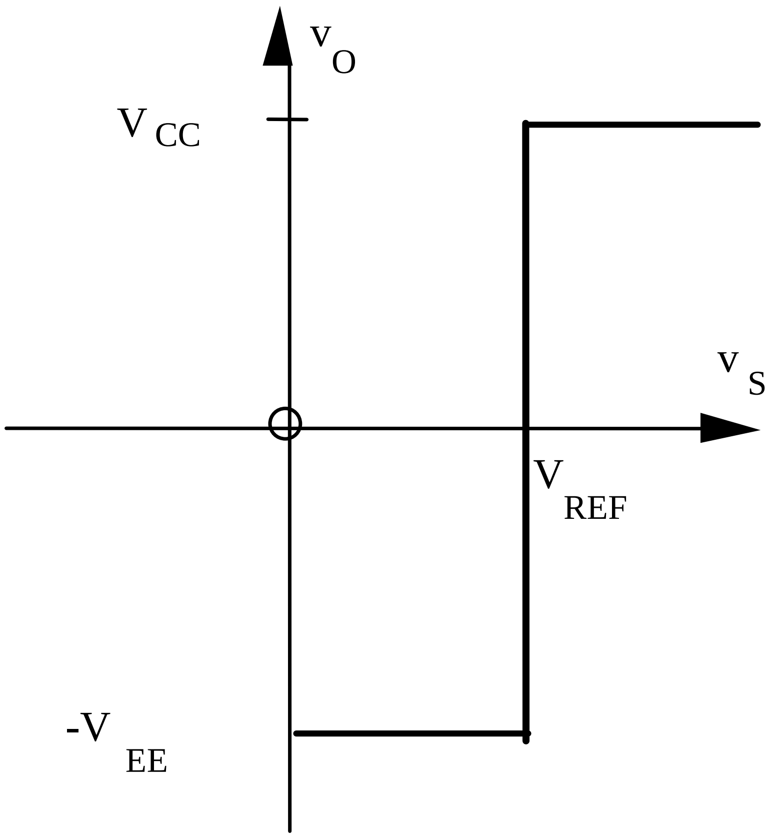
The Comparator and Schmitt Trigger (I)



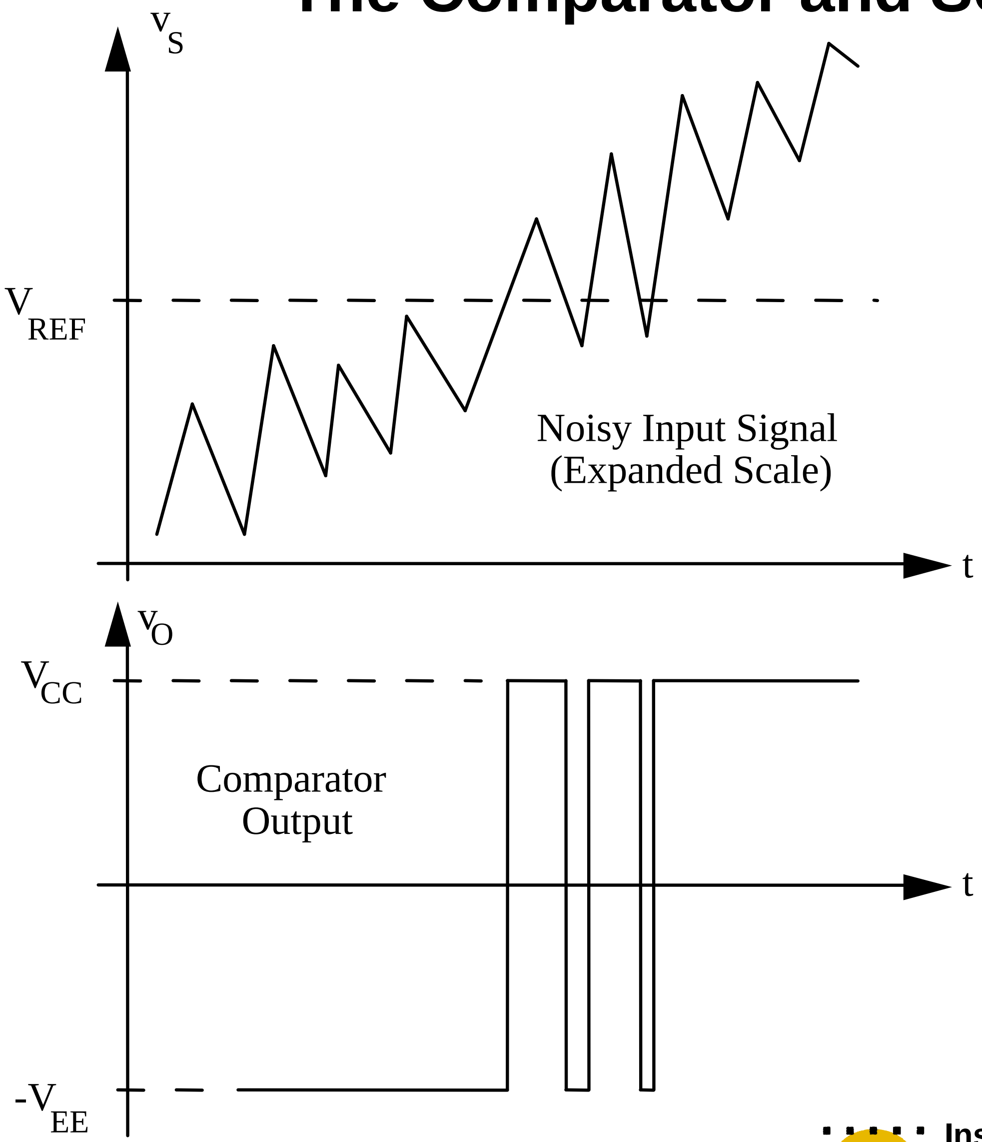
It is often useful to compare a voltage to a known reference level. This can be done electronically using the **comparator** circuit shown above.

The Comparator and Schmitt Trigger (II)

For input signals exceeding the reference voltage V_{REF}, the output saturates at V_{CC}; for input signals less than V_{REF}, the output saturates a $-V_{EE}$, as indicated in the voltage transfer characteristic shown on the right.



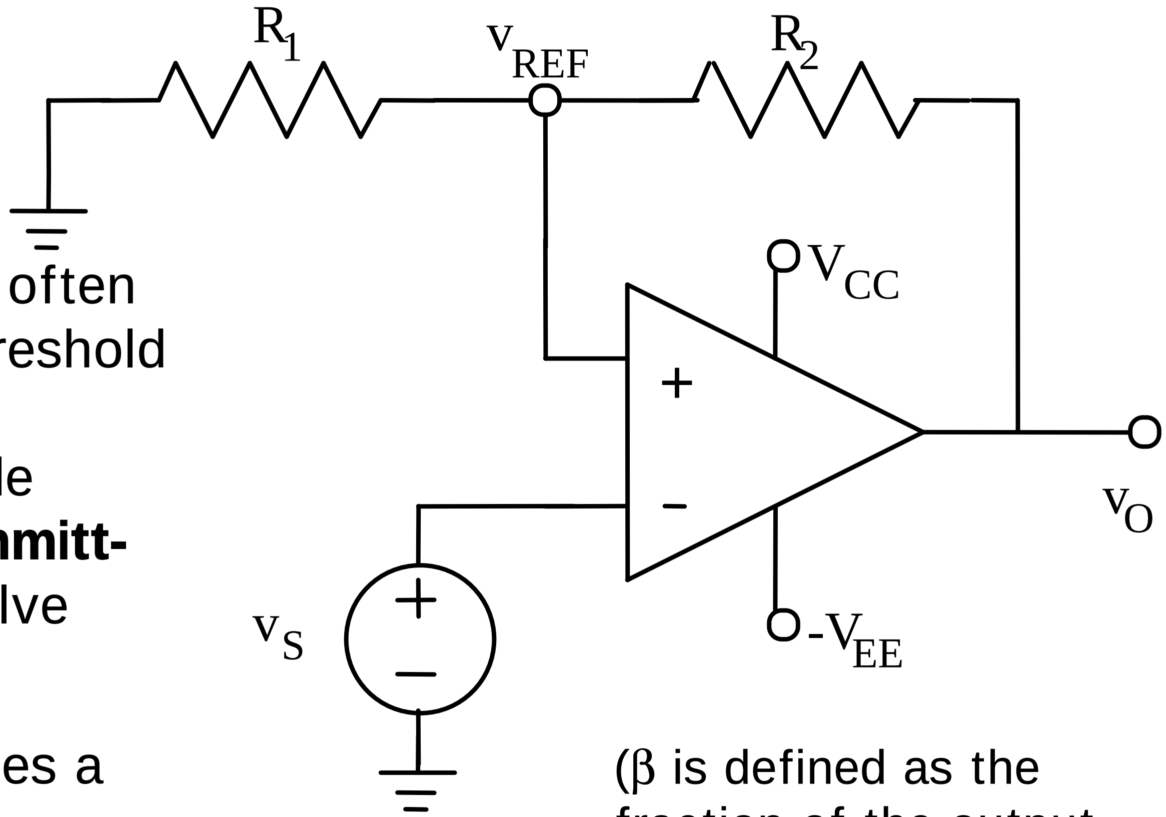
The Comparator and Schmitt Trigger (II)



However, a problem occurs when high-speed comparators are used with noisy signals.

As the input signal crosses the reference level, multiple transitions may occur due to the noise present on the input.

The Comparator and Schmitt Trigger (III)



In digital systems, we often want to detect this threshold crossing cleanly by generating only a single transition, and the **Schmitt-trigger** circuit helps solve this problem.

The Schmitt trigger uses a comparator whose reference voltage is derived from a voltage divider across the output (positive feedback).

$$b = \frac{R_1}{R_1 + R_2}$$

(β is defined as the fraction of the output voltage that is fed back from the output to the input and called the feedback factor)

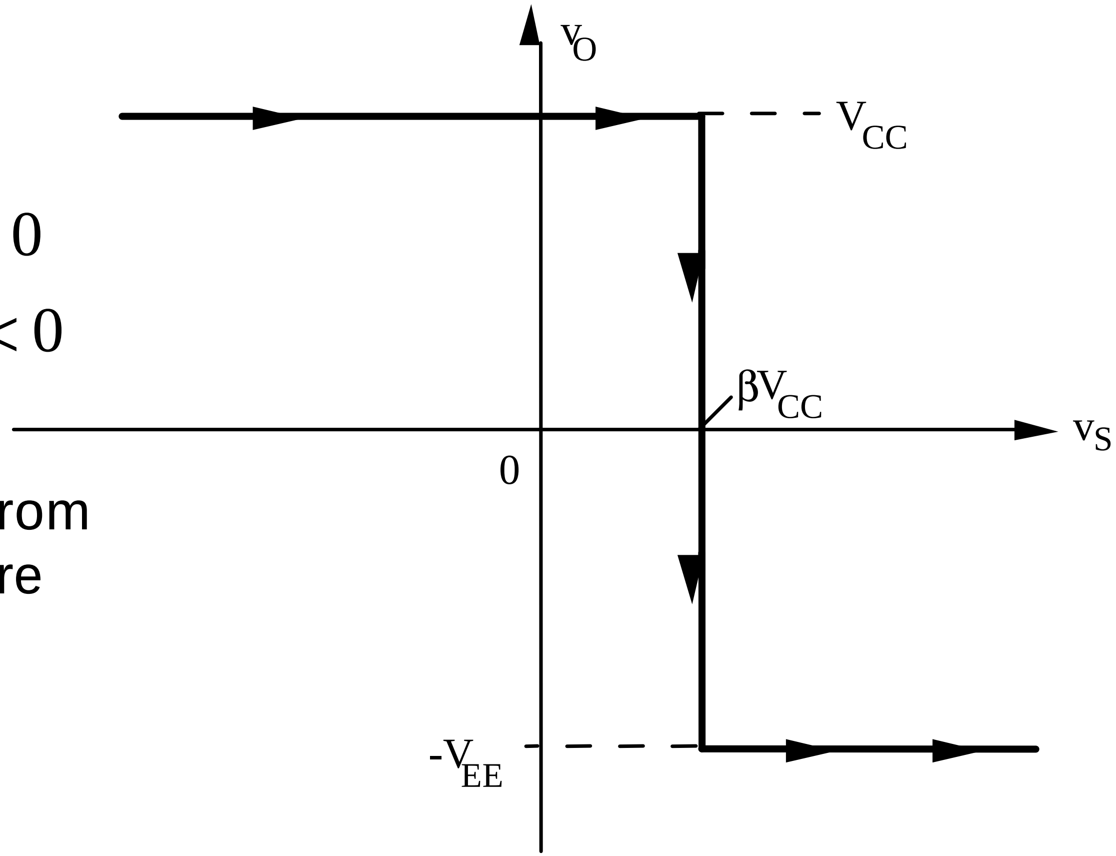
The Comparator and Schmitt Trigger (IV)

The reference voltage changes when the output switches state:

$$V_{REF} = \begin{cases} bV_{CC} & \text{for} \quad v_o > 0 \\ -bV_{EE} & \text{for} \quad v_o < 0 \end{cases}$$

Consider the case for an input voltage increasing from below V_{REF}, as in the figure on the right hand.

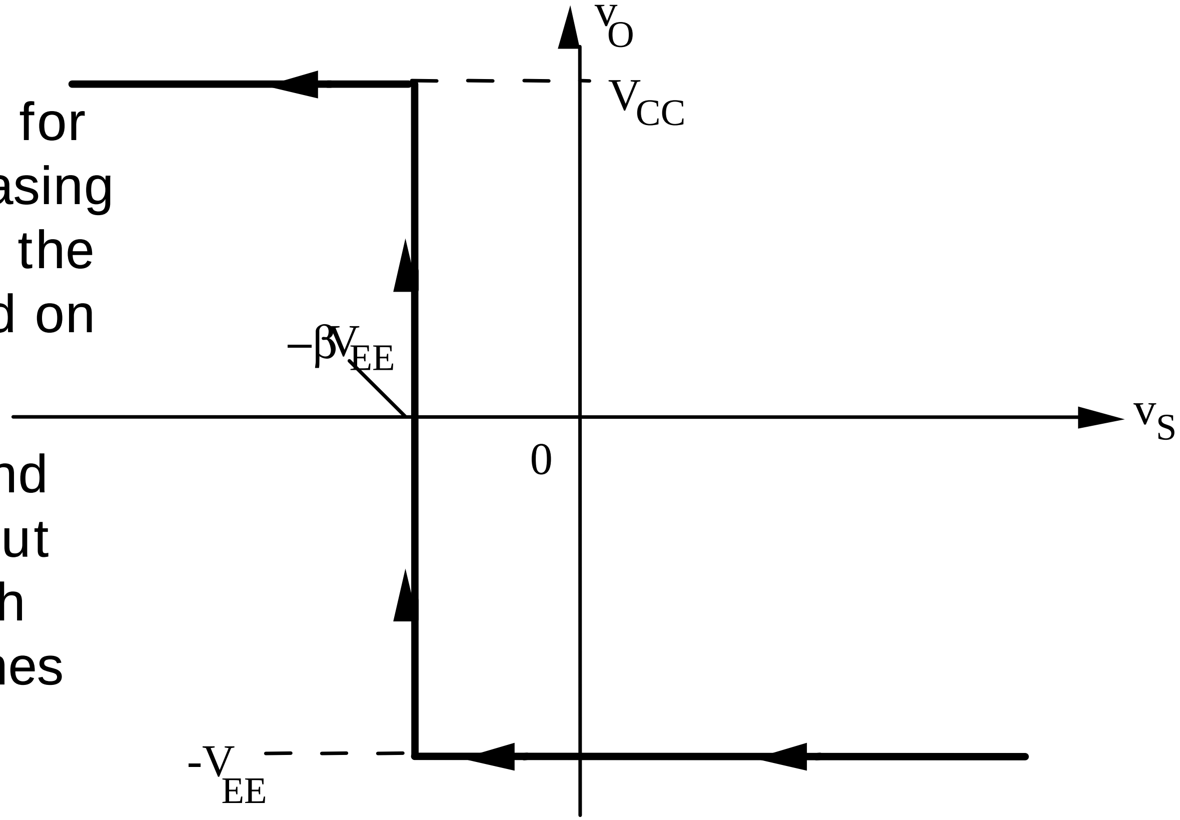
The output is at V_{CC} and $V_{REF} = bV_{CC}$. As the input voltage crosses through V_{REF}, the output switches state to $-V_{EE}$.



The Comparator and Schmitt Trigger (V)

Now consider the case for an input voltage decreasing from a high level, as in the figure on the right hand on this slide.

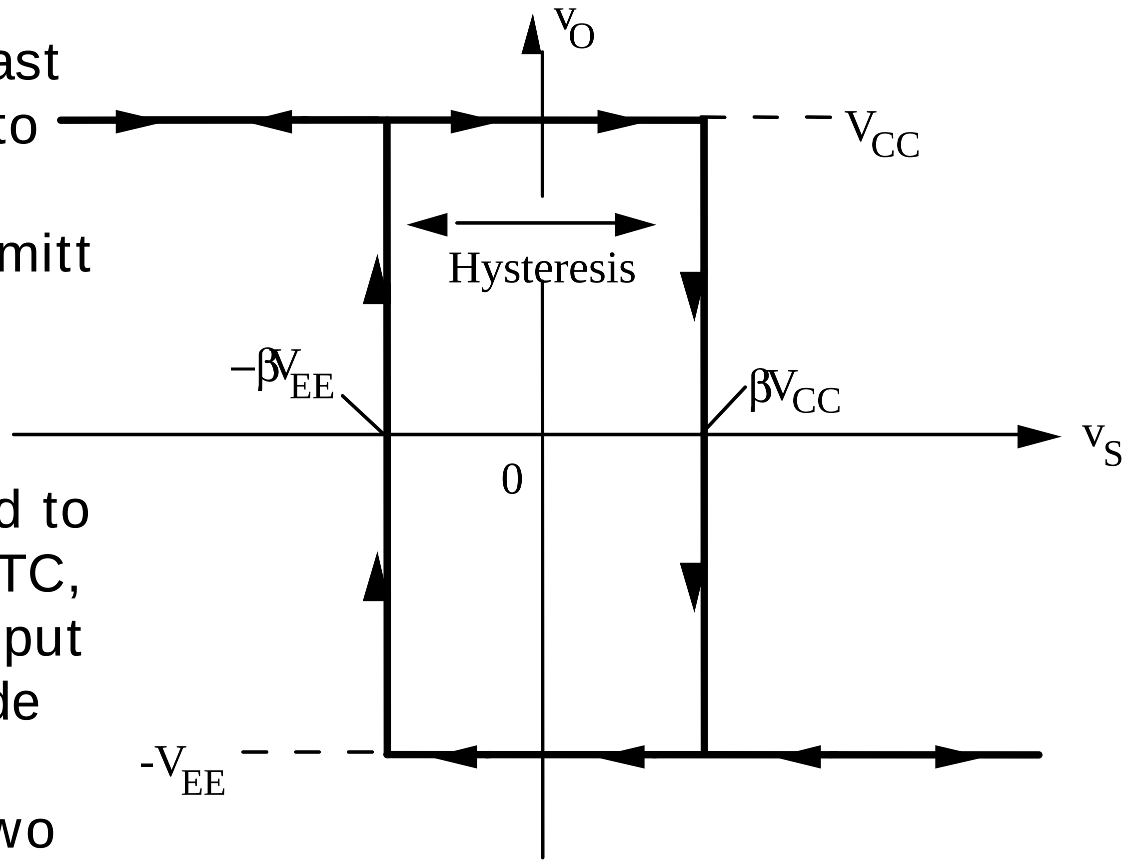
The output is at $-V_{EE}$ and $V_{REF} = -bV_{EE}$. As the input voltage crosses through V_{REF}, the output switches state to V_{CC}.



The Schmitt trigger with positive feedback is an example of an circuit with two stable states: a **bistable circuit**,or **bistable multivibrator**.

The Comparator and Schmitt Trigger (VI)

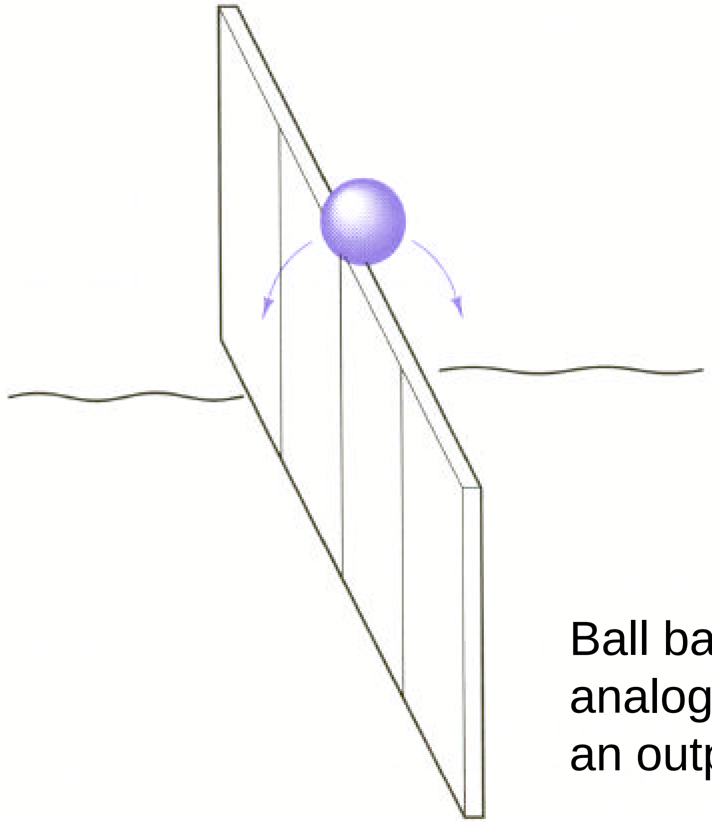
The voltage transfer characteristics from the last two slides are combined to yield the overall characteristic for the Schmitt trigger given here.



The Schmitt trigger is said to exhibit **hysteresis** in its VTC, and will not respond to input noise that has a magnitude V_N smaller than the difference between the two threshold voltages:

$$V_N < b\left[V_{CC} - (-V_{EE})\right] = b\left(V_{CC} + V_{EE}\right)$$

Bistable Circuits



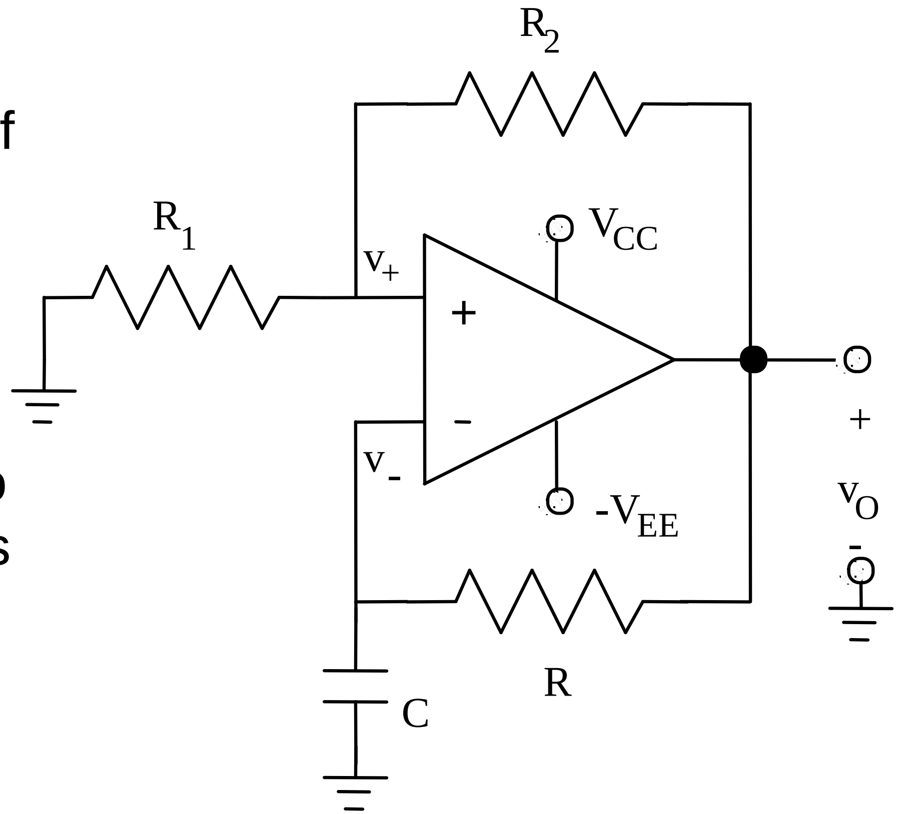
Ball balanced on top of fence is analogous to a Schmitt trigger with an output voltage of zero

MES Institute of Microelectronic Systems

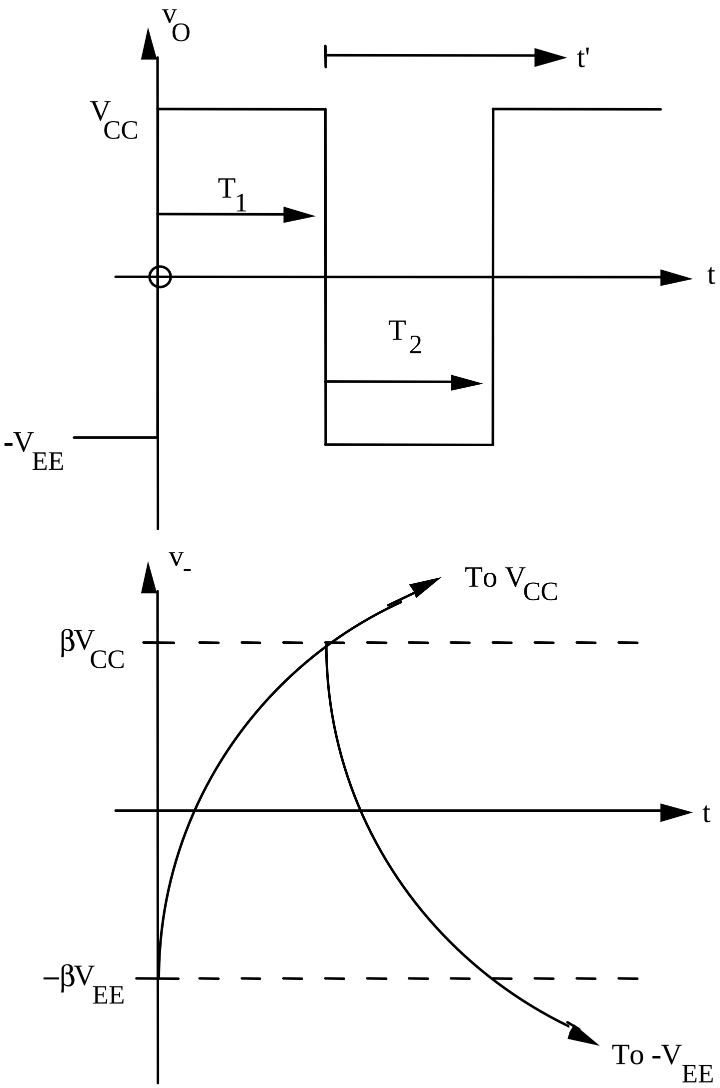
The Astable Multivibrator (I)

Another type of multivibrator circuit employs a combination of positive and negative feedback and is designed to oscillate and generate a rectangular output waveform.

The output of this circuit has no stable state and is referred to as an **astable circuit**, or **astable multivibrator**.



The Astable Multivibrator (II)



The output voltage switches periodically (oscillates) between the two output V_{CC} and $-V_{EE}$.

Let us assume that the output has just switched to $v_o = V_{CC}$ at $t = 0$. The voltage at the inverting-input terminal of the op amp charges exponentially toward a final value of V_{CC} with a time constant $t = RC$. The voltage on the capacitor at the time of the output transition is $v_C = -bV_{EE}$. Thus:

$$v_C(t) = V_{CC} - (V_{CC} + bV_{EE})e^{-\frac{t}{RC}}$$

The Astable Multivibrator (III)

The comparator changes state again at time T_1 when $v_c(t)$ just reaches bV_{CC}:

$$bV_{CC} = V_{CC} - (V_{CC} + bV_{EE})e^{-\frac{T_1}{RC}}$$

Solving for T_1 yields:

$$T_1 = RC \ln \frac{1 + b\left(\frac{V_{EE}}{V_{CC}}\right)}{1 - b}$$

The same procedure during time interval T_2 yields:

$$v_C(t') = -V_{EE} + (V_{EE} + bV_{CC})e^{-\frac{t'}{RC}}$$

The Astable Multivibrator (IV)

and

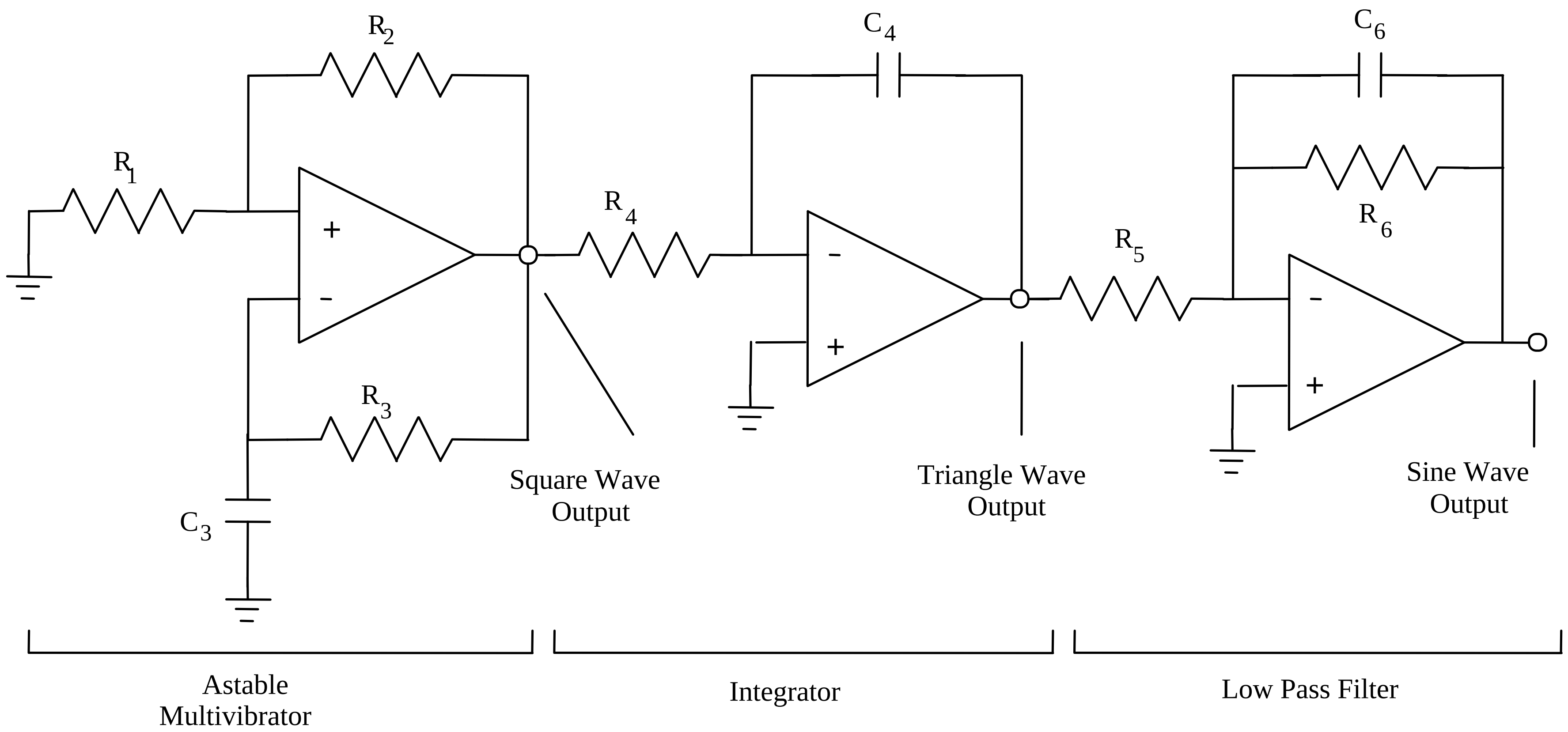
$$T_2 = RC \ln \frac{1 + b\left(\frac{V_{CC}}{V_{EE}}\right)}{1 - b}$$

And finally for the common case of symmetrical power supply voltages $V_{CC} = V_{EE}$:

$$T = T_1 + T_2$$

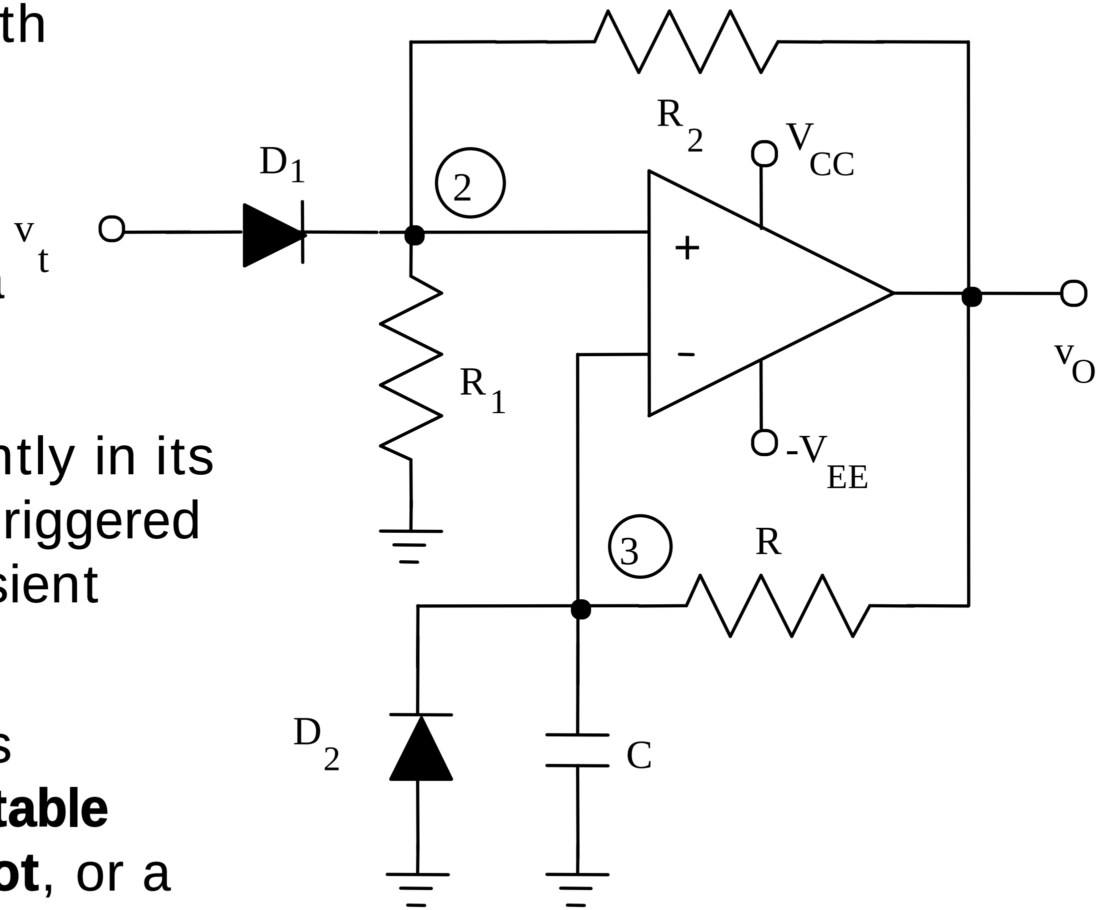
$$\boxed{T = 2RC \ln \frac{1 + b}{1 - b}}$$

The Astable Multivibrator (V): Application as an inexpensive function generator



The Monostable Multivibrator or One Shot (I)

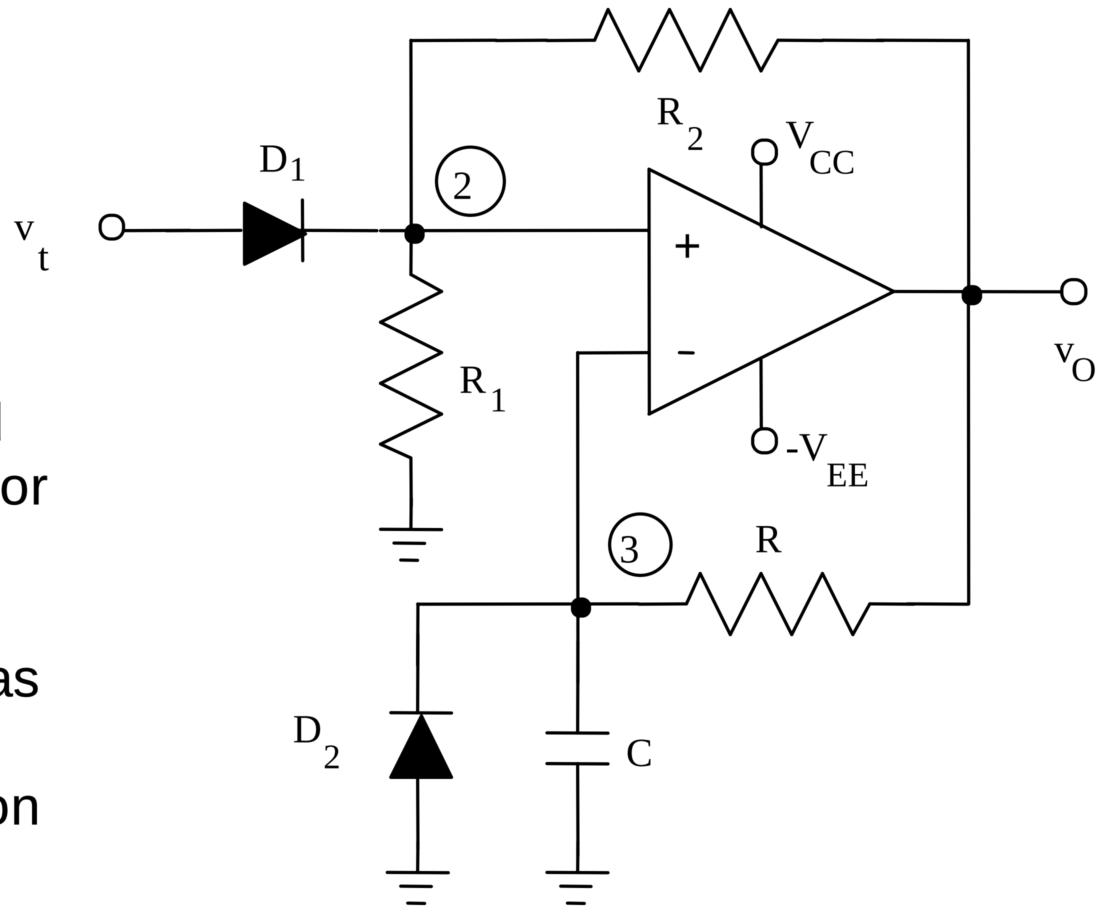
A third type of multivibrator operates with one stable state and is used to generate a single pulse of known duration following application of a trigger signal.



The circuit rests quiescently in its stable state, but can be triggered to generate a single transient pulse of fixed duration *T*.

This **monostable circuit** is variously called a **monostable multivibrator**, a **single shot**, or a **one shot**.

The Monostable Multivibrator or One Shot (II)



Diode D_1 has been added to the astable multivibrator to couple the triggering signal v_T into the circuit, and clamping diode D_2 has been added to limit the negative voltage excursion on capacitor C.

The Monostable Multivibrator or One Shot (III)

The circuit rests in its quiescent state with $v_o = -V_{EE}$. If the trigger signal voltage v_T is less than the voltage at node 2,

$$v_T < -\frac{R_1}{R_1 + R_2} V_{EE} = -bV_{EE}$$

diode D_1 is cut off. Capacitor C discharges through R until diode D_2 turns on, clamping the capacitor voltage at one diode-drop V_D below ground potential. In this condition, the differential-input voltage v_{ID} to the comparator is given by:

$$v_{ID} = -bV_{EE} - (-V_D) = -bV_{EE} + V_D$$

As long as the value of the voltage divider is chosen so that

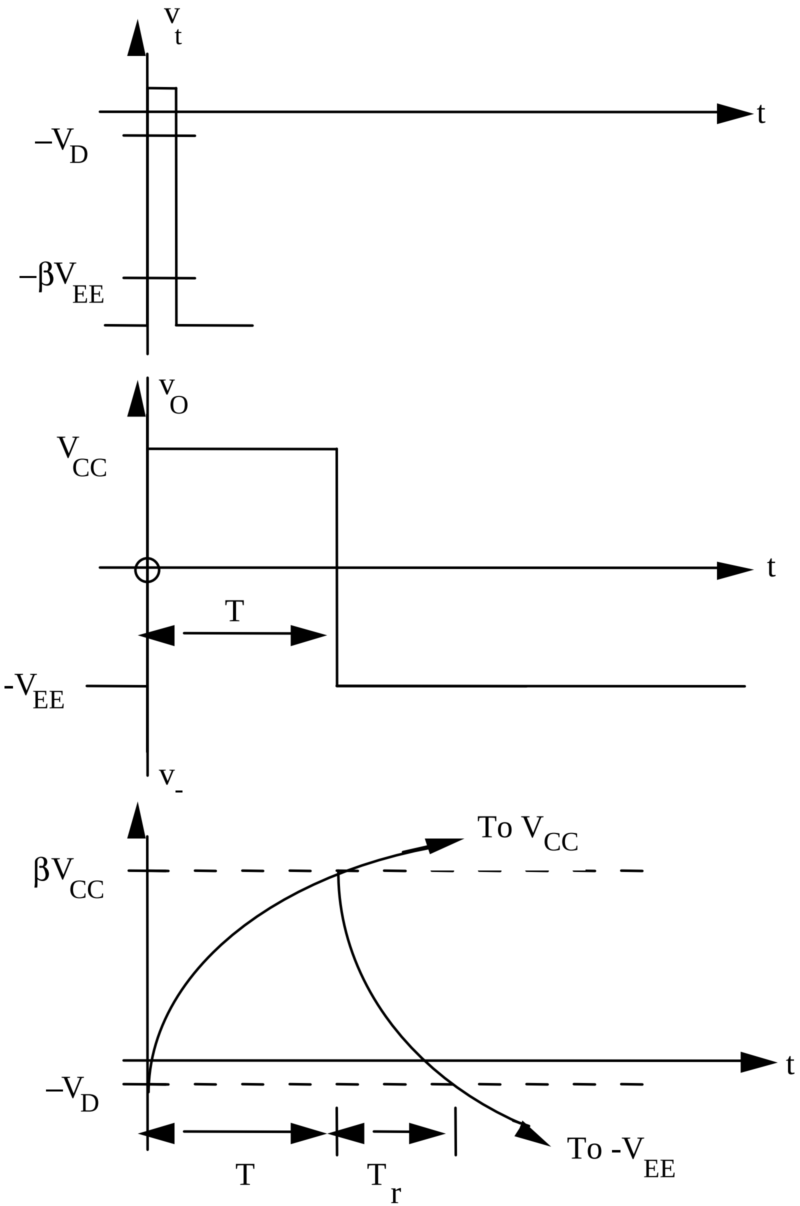
$$v_{ID} < 0 \quad \text{or} \quad bV_{EE} > V_D \qquad \text{where} \quad b = \frac{R_1}{R_1 + R_2}$$

then the output of the circuit will have one stable state.

The Monostable Multivibrator or One Shot (IV)

The monostable multivibrator can be triggered by applying a positive pulse to the trigger input.

As the trigger pulse level exceeds a voltage of $-bV_{EE}$, diode D_1 turns on and subsequently pulls the voltage at node 2 above that of node 3. At this point, the comparator output changes state, and the voltage at the noninverting-input terminal rises abruptly to a voltage equal to $+bV_{CC}$. Diode D_1 cuts off, isolating the comparator input from any further changes on the trigger input.



The Monostable Multivibrator or One Shot (V)

The voltage on the capacitor now begins to charge from its initial voltage $-V_D$ toward a final voltage of V_{CC} and can be expressed mathematically as

$$v_c(t) = V_{CC} - (V_{CC} + V_D)e^{-\frac{t}{RC}}$$

where the time origin ($t = 0$) coincides with the start of the trigger pulse. However, the comparator changes state when the capacitor voltage reaches $+bV_{CC}$. Thus, the pulse width T is given by

$$bV_{CC} = V_{CC} - (V_{CC} + V_D)e^{-\frac{T}{RC}}$$

or

$$\boxed{T = RC \ln \frac{1 + \left(\frac{V_D}{V_{CC}}\right)}{1 - b}}$$

Ideal Operational Amplifier (Summary)

The ideal operational amplifier actually has quite a number of additional implicit properties:

- Infinite common-mode rejection
- Infinite power supply rejection
- Infinite output voltage range (not limited by $-V_{EE} \leq v_O \leq V_{CC}$)
- Infinite output current capability
- Infinite open-lop bandwidth
- Infinite slew-rate
- Zero output resistance
- Zero input-bias currents and offset currents
- Zero input-offset voltage